Algorithm for TOA-based indoor geolocation

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A new indoor geolocation algorithm based on time-of-arrival (TOA) range measurements in a wireless network is presented. The algorithm, referred to as closest neighbour with TOA grid (CN-TOAG), is described and its performance compared with traditional least-squares (LS) methods.

Introduction: Emerging indoor geolocation systems use either geometric techniques [1, 2], or pattern recognition algorithms [3, 4] to locate a user. This is done in two stages. The first stage is to observe some characteristic of the transmitted signal, such as received signal strength (RSS), or delay (TOA) to derive location metrics. The second stage is to feed these metrics into a geolocation algorithm [1]. TOA-based systems use geometric methods, such as standard least-squares (LS) and the residual weighting LS (RWGH) algorithms.

In this Letter we present a new algorithm for accurate indoor geolocation using TOA, and compare its performance quantitatively with the traditional LS and RWGH algorithms. There are many different approaches to LS algorithms; in this Letter, an LS algorithm developed by Davidon [5] has been used. The RWGH algorithm used, described in [6], is a form of weighted LS algorithm. It has been proposed as a method of mitigating the effects of range measurement errors resulting from obstructed line-of-sight (OLOS) channel conditions.

CN-TOAG algorithm: In the CN-TOAG algorithm, we take advantage of the fact that for any given point in the area covered by a number of base stations (BSs), we know the exact value of the expected TOA from all the BSs. Consider the grid arrangement of BSs in an indoor setting, as shown in Fig 1. Each of these BSs would perform a range measurement, d_i ($1 \leq i \leq N$, where N is the number of BSs in the grid) to the user to be located.

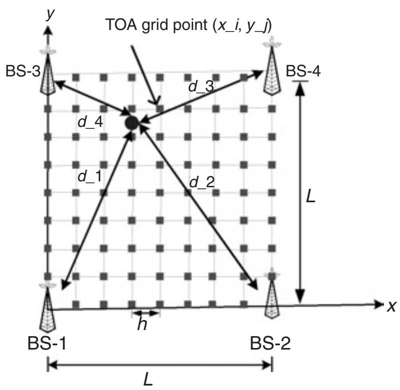

Fig. 1 *Geolocation system showing TOA grid for CN-TOAG algorithm*

Let **D** represent the vector of range measurements that are reported by the BSs, and let **Z** represent the vector of expected TOA-based range measurements at a certain point, $\mathbf{r} = (x, y)$. We call **Z** the *range signature* associated with the point **r**. An estimate of the user's location, $\hat{\mathbf{r}}$, can be obtained by finding that point **r**, where **Z** most closely approximates **D**. We define an error function, $e(\mathbf{r}) = e(x, y)$, as:

$$e(\mathbf{r}) = e(x, y) = \|\mathbf{D} - \mathbf{Z}(\mathbf{r})\| = \|\mathbf{D} - \mathbf{Z}(x, y)\| \tag{1}$$

where $\|\cdot\|$ represents the vector norm. Equation (1) can also be written as:

$$e(x, y) = \sqrt{\sum_{k=1}^{N} \left(d_k - \sqrt{(x - X_k)^2 + (y - Y_k)^2} \right)^2} \tag{2}$$

where N is the number of BSs, d_k is the range measurement performed by kth ($1 \leq k \leq N$) BS, and (X_k, Y_k) represents the location of the kth BS in Cartesian co-ordinates. The estimated location of the mobile, $\hat{\mathbf{r}}$, can then be obtained by finding the point (x, y) that minimises (2). This point can be found by using the gradient relation:

$$\nabla e(x, y) = \mathbf{0} \tag{3}$$

i.e.

$$\hat{\mathbf{r}} = \operatorname{argmin}(\|\mathbf{D} - \mathbf{Z}(\mathbf{r})\|) \tag{4}$$

where argmin($\cdot$) refers to the value of the location co-ordinates that minimises the error function. Owing to the complexity of the function in (2), it is not possible to find an analytical solution to this problem. Therefore, we have developed a numerical method, which we refer to as the CN-TOAG algorithm.

With the CN-TOAG algorithm, the area covered by the BSs is divided into equally sized cells, using grid points as shown in Fig. 1. Each grid point is located h metres away from its immediate horizontal and vertical neighbours, as shown in Fig 1. Each grid point, (x_i, y_j), has a range signature, $\mathbf{Z}(x_i, y_j)$, associated with it. It must be noted that the range signature associated with each grid point is exact, since it is based on straightforward geometric calculations. The CN-TOAG algorithm estimates the location of the user as the grid point where the range signature most closely approximates **D**. To find this point, the CN-TOAG algorithm calculates the value of the error function, e_{ij}, at each point on the TOA grid:

$$e_{ij} = e(x_i, y_j) = \|\mathbf{D} - \mathbf{Z}(x_i, y_j)\| \tag{5}$$

The CN-TOAG algorithm then gives the estimated location, $\hat{\mathbf{r}}_{ij} = (\hat{x}_i, \hat{y}_j)$, as the location with the minimum e_{ij}. As can be gathered from the preceding discussion, the granularity of the TOA grid, as embodied in the parameter h, is a major determinant of performance for this algorithm.

Performance evaluation: Accurate models of range measurement error in the indoor environment are necessary for performance evaluation of algorithms. Recently, models characterising the statistics of range measurement errors in line-of-sight (LOS), and obstructed LOS (OLOS) channel conditions have been reported in the literature [7]. The results reported in this Letter use those models.

The performance comparison among CN-TOAG, LS and RWGH algorithms is assessed through simulations. A grid arrangement of four base stations is assumed to cover a 20 × 20 m area, as shown in Fig. 1. System bandwidth values of 50, 500 and 1000 MHz are considered for the channel models. The independent parameter in all cases is the grid granularity, as given by h. All results are presented for the case of the OLOS channel scenario. The performance metric is the root-mean-square positioning error ($RMSE_{pos}$), defined as:

$$RMSE_{pos} = \sqrt{E\{(|\hat{\mathbf{r}} - \mathbf{r}|)^2\}} \tag{6}$$

where **r**, and $\hat{\mathbf{r}}$ are the actual and estimated locations of a user, respectively.

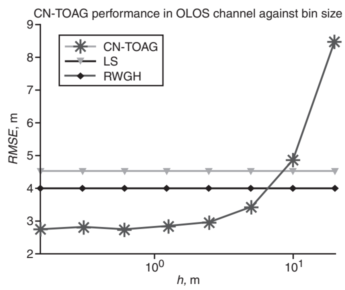

Fig. 2 *Comparison of CN-TOAG performance against LS and RWGH algorithms*

20 × 20 m area, system bandwidth = 500 MHz

The results of these simulations are shown in Figs. 2 and 3. Fig. 2 shows the comparison of the CN-TOAG algorithm with LS and RWGH algorithms, using a system bandwidth of 500 MHz. As shown in Fig. 2, CN-TOAG can achieve a better performance than the LS algorithm for our system scenario, provided that $h < 8.5$ m. We can see from Fig. 2 that while the LS algorithm has an *RMSE* of about 4.5 m, CN-TOAG has an *RMSE* value that can go down as much as 2.75 m (which implies a 38% improvement in estimation accuracy). We also observe from Fig. 2 that CN-TOAG performs better than the RWGH algorithm, provided that $h < 6.5$ m. Specifically, we see that the *RMSE* can go down from 3.14 m (for RWGH) to as much as 2.75 m in the case of CN-TOAG, reflecting a 12% improvement in performance.

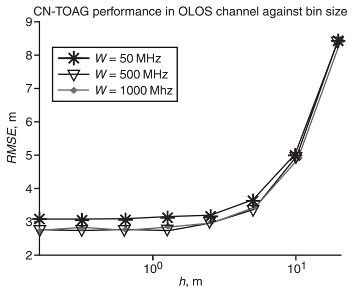

Fig. 3 *CN-TOAG performance at various bandwidth values*
20 × 20 m area

Fig. 3 depicts the performance of CN-TOAG at system bandwidth values of 50, 500 and 1000 MHz. From Fig. 3, we observe that CN-TOAG performance does not appear to improve appreciably beyond a certain value of h. For the system scenario considered for Fig. 3, this value of h is about 1.25 m. Furthermore, we note that CN-TOAG performance essentially stays the same between system bandwidth values of 500 and 1000 MHz. As the bandwidth of the system used to make the range measurements is increased, the range measurements themselves would be more accurate. This is normally expected to translate to a more accurate location estimate. However, for the CN-TOAG algorithm, the results suggest that beyond a certain point, increasing the system bandwidth any further will not necessarily result in greater accuracy in the location estimates.

Conclusions: We have introduced a new algorithm, CN-TOAG, for indoor positioning, and have compared its performance with LS and RWGH algorithms. We have shown that for the four BS scenarios considered in this Letter CN-TOAG performs 38 and 12% better than LS and RWGH, respectively. We have also shown that CN-TOAG performance is relatively insensitive to both the grid granularity (1.25 m in our scenario) and system bandwidth beyond a certain point (500 MHz in our scenario).

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