where

$$z = \sup_{0 \leq x \leq 1} \left[ \left( \frac{1}{p_1 - p_2} \right) \left| (p_1 e^{-pt} + p_2 e^{-2pt}) k_1 \right| \delta(t) + (p_1 k_1 e^{-pt} - p_2 k_1 e^{-2pt}) \right]$$

(9)

However, this inequality implies that

$$|\epsilon_k(t)| \leq |\delta_k(t)|$$

and therefore that

$$|\epsilon_k(t)| \leq |\delta_k(t)| k^!$$

(10)

$$|\epsilon_k(t)| \leq |\delta_k(t)| k^!$$

(11)

where

$$\beta = \sup_{0 \leq x \leq 1} |\delta_k(t)|$$

Hence

$$|\epsilon_k(t)| \to 0$$

(12)

as \(k \to \infty\), which implies that

$$\epsilon_k(t) \to 0$$

uniformly in \(t \in [0, T]\) as \(k \to \infty\). However, it follows from (iii) and (iv) that

$$\epsilon_k(t) \to 0$$

(13)

for all \(k = 0, 1, 2, \ldots\). It is therefore finally evident that

$$\epsilon_k(t) \to 0$$

uniformly in \(t \in [0, T]\) as \(k \to \infty\).

**Illustrative example:** This theorem can be conveniently illustrated by considering the iterative learning control of the second-order plant governed by eqn. 1 with \(a_1 = 2, a_2 = 1\) and \(\gamma = 1\), which is completely irregular and therefore cannot be controlled directly by Arimoto et al.\(^2\) In the case \(\epsilon_0(0) = 0\), the rapid learning of the controller with \(k_1 = 1\) and \(k_2 = 1\) is shown in Fig. 1. In this case, \(\epsilon(t)\) is such that \(\epsilon_k(0) = 0\) for all \(k = 0, 1, 2, \ldots\), and therefore no shifting of the initial velocity is required to satisfy condition (iv) of the theorem.

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**DOPPLER SPREAD MEASUREMENTS OF INDOOR RADIO CHANNEL**

**Indexing terms:** Radiowave propagation, Radiocommunication

The temporal variations caused by human movement or human-induced motion of the communication equipment, on the otherwise static indoor radio channel, are measured. For the reported narrowband measurements, a maximum Doppler spread of 6.1 Hz and a maximum RMS Doppler bandwidth of 0.87 Hz are observed.

**Introduction:** The path length changes in a fading multipath channel give rise to Doppler shifts, and the width of the distribution of Doppler shifts is defined as the Doppler 'spread'. The Doppler spread is the width of the received spectrum when a single tone is transmitted, and it is related to the rate at which fading occurs. The Doppler spread is important in determining the minimum signalling rate allowable for coherent demodulation and the minimum adaptation rate for an adaptive receiver.\(^1\) For indoor radio channels, the multipath characteristics are static at any one location until some movement occurs and causes Doppler spreading.

This letter presents calculations of RMS Doppler bandwidth and Doppler spread of the indoor radio channel caused by traffic and local movements of the communication equipment. These were controlled experiments, with the only movement being the movement for which we were trying to determine the resulting Doppler spread. The measurements were divided into line of sight (LOS) and obstructed line of sight (OLOS).\(^2\) The measurements were taken on the third floor of the three-storey Atwater Kent Laboratories at the Worcester Polytechnic Institute.

**Measurements:** A 910 MHz signal generated by a network analyser is used as the input to a 45 dB transmitter RF amplifier. The output of the RF power amplifier is propagated by a dipole antenna. The signal from the receiver dipole antenna is passed through an attenuator, a series of amplifiers with a gain of 60 dB, and is returned to the network analyser to determine the time variations of the channel. In a 32 s interval, the network analyser takes samples of the amplitude and phase of the received signal at a rate of 25 samples/s.

Table 1 lists the distance between the transmitter and the receiver for the 11 LOS measurements performed in the electronics laboratory. For reference purposes, measurement 1 was taken at 1 m with no movements. Three measurements at each of the three distances of 3, 6 and 12 m of receiver/transmitter separation were taken. Measurements 2, 5 and 8 were taken with people moving in the path between the transmitter and receiver. Measurements 3, 6 and 9 were taken with human traffic close to the transmitter and receiver. Measurements 4, 7 and 10 were taken with small cyclic movement of the transmitter. Measurement 11 was taken at 12 m separation with random motion of the transmitter.

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**References**

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For the OLOS measurements, the receiver was placed in the communication research laboratory (CRL). A total of 10 measurements, summarised in Table 1, were taken in three different transmitter locations. Location 1 was in the corridor next to the CRL, location 2 was in the microwave laboratory at the other side of the corridor; and location 3 was in the electronics laboratory next to the microwave laboratory. Measurements 4 and 7 were taken with human traffic close to the transmitter. Measurements 2, 5 and 8 were taken with human traffic close to the receiver. Measurements 3, 6 and 9 were taken with small cyclic movement of the transmitter. Measurement 10 was taken at the third location with traffic in between the transmitter and receiver, but not in the same room as either the transmitter or the receiver.

<table>
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<tr>
<th>No.</th>
<th>Dis.</th>
<th>$f_c$</th>
<th>$B_o$</th>
<th>Power</th>
<th>Dis.</th>
<th>$f_c$</th>
<th>$B_o$</th>
<th>Power</th>
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Table 1: $f_c$ AND $B_o$ IN Hz, DISTANCE IN m AND POWER VARIATION IN dB

Analysis of data: In the analysis of acoustic or radar signals, the maximum Doppler shift is related to the velocity of movement $v_o$ at the transmitting frequency $f_o$ by

$$f_o = \frac{2\pi v_o}{c}$$

where $c$ is the velocity of propagation. If this equation is used as an approximation to the Doppler spread $B_o$ of the indoor radio channel, a person walking at 3 m/s (1.33 m/s) will produce a Doppler spread of 8 Hz for the 910 MHz transmission frequency. In the rest of this section the exact Doppler spreads of the previously described measurements are determined.

Our measurements give the response $H(f, \lambda)$ of the indoor radio channel to an unmodulated sine wave of frequency $f_o = 910$ MHz. The complex autocorrelation function of $H(f, \lambda)$ is $R(f, \lambda)$, and the Doppler power spectrum is

$$V(f, \lambda) = R(f, \lambda) \exp(-j2\pi f \lambda)$$

Since $V(f, \lambda)$ is the Fourier transform of the complex autocorrelation function $H(f, \lambda)$, $V(f, \lambda) = |H(f, \lambda)|^2$, where

$$H(f, \lambda) = \int H(f, \lambda) \exp(-j2\pi f \lambda) df$$

The Doppler spread $B_o$ is the range of wavelengths $\lambda$ over which the Doppler power spectrum $V(f, \lambda)$ is nonzero. For real measurements, $V(f, \lambda)$ is never zero, and a threshold at -10 dB is used to determine $B_o$. A more specific measure of the Doppler spread is the RMS Doppler bandwidth, given by

$$B_o = \sqrt{\frac{\int V(f, \lambda) df}{\int V(f, \lambda) df}}$$

which is a measure of the distribution of the power rather than just the width of the power distribution.

Fig. 1a shows OLOS measurement 1 with human traffic close to the transmitter. Fig. 1c shows OLOS measurement 9 for cyclic motion of the transmitter. Table 1 gives both $B_o$ and $f_c$ for all the measurements. Except for LOS measurement 1, which was taken with no movement, the $f_c$ fall in the range 0.1–9.9 Hz and the $B_o$ are less than 6.1 Hz. The cyclic motion of the transmitter caused power variations of less than 10 dB, and movement of people caused power variations of up to 35 dB for both LOS and OLOS measurements.
GaAs W-BAND IMPATT DIODES FOR VERY LOW-NOISE OSCILLATORS

Introduction: It has recently been shown that single-drift flat-profile GaAs impatt diodes successfully operate up to W-band frequencies with sufficiently high output power (270mW typically) and efficiency (6% typically). These diodes on diamond heat-sinks are fabricated from MBE material using a selective etching technology to achieve reproducible results. W-band single-drift flat-profile impatt diodes were fabricated from GaAs MBE material and tested in a full-height waveguide resonant cavity with resonant cap. A quasi-optical parabolic Fabry-Perot resonator was used to determine the FM noise of the GaAs impatt oscillator. With a minimum noise measure of 20dB at power levels around 20mW, impatt diode oscillators can compete well with oscillators using Gunn devices. The (V/1) min = 82dBc measured at 100kHz frequency off-carrier and at Qex = 95 is comparable to the value obtained from Gunn devices. The maximum available output power of 270mW, however, markedly exceeds that of Gunn oscillators.

Measurement set-up: The complete measurement set-up is depicted in Fig. 1. As a versatile tool to convert the FM into AM, a quasi-optical parabolic Fabry-Perot resonator is used. This resonator is similar to the system described in References 4 and 5 for W-band and D-band applications. A detector at the output of the resonator delivers the demodulated FM signal. The low-noise amplifier decouples the signal of the detector diode and improves the sensitivity of the detector in the same way as discussed in Reference 5. The tracking generator of the selective level meter is used to determine the sensitivity of the Fabry-Perot resonator with a standard Bessel zero method at the test signal frequency f0. The sensitivity S = u0/(2(1240f0)) for a demodulated signal u0 at the detector lies between 3 and 15mV/Hz, depending on the input power. The frequency and amplitude of the modulating Bessel zero signals are kept constant during one computer-aided measurement cycle. This signal is switched to the diode between each measuring point, where the offset frequency f0 changes. Using this signal, both a small drift in the modulation index of the impatt oscillator can be seen or the spectrum analyser and, more importantly, a drift in the sensitivity of the quasi-optical resonator (caused by mechanical drift or drift of the oscillator frequency) can be taken into account by the computer program. Additional errors thus caused by drift effects can be excluded within the run, i.e. for all the noise levels at frequencies off-carrier between 2kHz and 1.7 MHz.

The external quality factor Qex of the impatt oscillator is determined by the self-injection locking method, for which the first 20dB coupler in Fig. 1 with the attenuator and a tunable short is used.

Results: Fig. 2 shows the typical FM noise deviation Δf0n for two different diodes and power levels plotted as a function of the frequency f0 off-carrier. A detector against frequency off-carrier f0.

The single-sideband FM noise deviation Δf0n of typical GaAs single-drift flat-profile impatt diodes against frequency off-carrier f0.

- f0 = 94.5GHz, P = 22.5mW, Qex = 95
- f0 = 89.9GHz, P = 9.8mW, Qex = 80

The single-sideband FM noise deviation (V/1)min at a frequency off-carrier f0 can be calculated from the FM fre-

Fig. 1 Block diagram of noise measurement set-up with quasi-optical resonator

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