Performance Bounds of Multi-Relay Deployment of Wireless Data Networks

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Abstract—In a multi-rate wireless data network, the data rate is a function of the distance from the Access Point (AP). To extend the coverage range of the data network and to increase the throughput to a target terminal outside the AP coverage range, relay nodes are inserted between the AP and the target terminal. Therefore, for a given throughput-distance relationship and a given distance between the access point and the target receiver, there must be a minimum number of nodes that provides the maximum throughput to the target receiver. In this paper we present an analytical approach to determine the optimal performance bounds and the number of relays for the optimal deployment of a multi-hop multi-rate wireless data network. The optimal number of relays is chosen to optimize the throughput for two scenarios. The first is to maximize the spatial throughput up to a desired coverage distance. The second is to maximize the throughput for a target receiver located at a specific distance from the access point.

I. INTRODUCTION

Recently, there has been significant interest in performance analysis of multi-hop multi-relay wireless networks [1]. The primary focus is on wireless sensor network applications. The main focus is on the energy-throughput relationship where both the physical (PHY) and the Medium Access Control (MAC) layers are considered in the performance analysis and optimal selections of the number of hops [2]. On the other hand, wireless data network deployments are concerned with the maximization of the throughput to a target receiver located at some distance from the primary Access Point (AP). Deployment of data networks is gaining attention with the introduction of new wireless transmission standards such as 802.11ac [2] [3] [4].

When a target receiver is located beyond the coverage range of an AP, relay nodes are deployed between the source and destination terminals to extend the coverage. In some situations, relays are used within the coverage area of the AP to increase the throughput after a certain distance from the AP at the expense of requiring multiple transmissions for a single packet. In a multi-rate transmission data network such as a Wireless Local Area Network (WLAN), the data rate is a function of the distance from the access point or the relay. Therefore, for a given relationship between the throughput and the distance and a given distance between the access point and the target receiver there must be a minimum number of relays that provides the maximum throughput.

Using empirical data from simulation platforms, the impact of a multi-hop wireless channel on the performance of a WLAN has been discussed in the literature [5] [6] [7]. In these analyses, however, the transmission data rate of an AP is assumed to be fixed and hops between the transmitter and the receiver are managed so that they can carry the load provided by the AP. In other words, the links between the AP and the relays along with the links between the relays all share the same performance capabilities. In practice, all modern wireless data communication networks such as WLANs, WPANs and cellular data services are implemented on multi-rate modems. They are installed in scenarios where the distances between different elements of the network are not the same. Since in multi-rate modems the data rate is a function of distance between the transmitter and the receiver [8], the throughput of a link must be a function of the distance as well. In particular, for a realistic installation with different link lengths among the elements of the network we are interested in bounds on the performance of the network. Results of both empirical measurements [9] [10] [11] and analytical modeling [12] [13] [14] of the relation between the throughput and distance in a WLAN reveal that this relationship can be shown by a continuous function of distance that is parameterized by the maximum possible throughput and decay rate of the first link. On the other hand, results of empirical data for multi-hop networks [5] [6] [7] reveal that the Medium Access Control (MAC) throughput in a multi-hop network with N-hops is proportional to the maximum physical (PHY) layer throughput that occurs without relays. Optimization of the deployment of multi-hop multi-relay networks has been investigated in the literature. The optimal number of hops for IEEE802.11 RTC/CTS MAC to maximize the throughput and minimize the energy consumption was investigated in [2]. The energy-throughput relationship included the effects of the PHY and MAC layers. The developed model assumed free space propagation to simplify the analysis. The optimal number of relays was selected to maximize the throughput and minimize the energy consumption for a given data rate. The placement of a single relay station (RS) problem in IEEE 802.16j networks was explored in [15], using a cooperative relay strategy of Decode-Forward (D-F) or Compress-Forward (C-F) to find both the optimal RS location and relay time allocation in a single stage. The aim was to obtain maximum overall system...
capacity as well as to meet the uneven distributed traffic demand of each subscriber station (SS). Multiple accesses with collision avoidance were used in WLANs, and hidden terminal (HT) and exposed terminal (ET) problems occurred depending on the distance between stations and the carrier sensing range. An AP cooperation system was proposed that detects the HT and ET problems between stations (STAs) [16]. Focusing on the uplink and categorizing the MAC-level problems that occur in densely deployed WLANs, the system integrated the observed information and then detected the MAC-level problems based on their connected APs. The simulation results showed that the system can detect the MAC-level problems accurately, regardless of AP density or shadowing effect. Using IEEE 802.16e module for NS-2, particularly throughput versus distance, results were obtained [17]. A quantitative analysis was conducted via extensive simulation, and the optimal modulation and coding scheme as well as channel bandwidth profiles were identified for specific distances. The obtained data was used in test bed designs intended for installation along the BNSF Railway track in Nebraska. Also, the client access coverage area provided is investigated by this module and its expected throughput was determined.

In this paper, we develop a mathematical framework for the analysis of the behavior of multi-hop data networks as function of the distance between the source terminal, destination terminal and the intermediate distances between the relays and AP. First, we determine the intermediate distances between the AP and relays to obtain the optimal performance bounds as a function of the distance from the AP to the target terminal and the number of relays. Then we determine the required number of relays to get the optimal throughput at a target terminal located at a desired distance. We demonstrate the results of the analysis using a general analytical model for the PHY throughput-distance relation. For the MAC layer, we use the IEEE 802.11 RTS/CTS protocol. Using the general PHY model we show the optimal performance bounds for the network and the MAC throughput distance relation for a deployment optimized for either coverage or specific distance.

The remainder of the paper is divided as follows: In Section II we present the network system model and general statement of the problem. In section III we show the proposed solution theorems and their proofs. Section III contains an application example for a general analytical PHY throughput-distance model. Section IV provides the conclusions of our research.

## II. Network System Model

### A. Physical Layer Model

In wireless transmission, the physical (PHY) layer throughput is a monotonically increasing function of the signal-to-noise ratio. In practice, manufacturer data sheets specify the data rate as a discrete function of the receiver sensitivity. Table I shows the general description of the discrete relationship between the PHY throughput and the receiver sensitivity. If we assume the target terminal T is located at a distance \( r_1 \) within the Access Point (AP) coverage range, the received signal power \( P_r \) is given by

\[
P_r(r_1) = g(r_1) + \eta (1)
\]

where \( g(r_1) \) is the path loss model used for the transmission medium and \( \eta \) is a Gaussian random variable with variance \( \sigma \) representing shadow fading. The IEEE 802.11 standard defines \( g(r_1) \) as

\[
g(r_1) = \begin{cases} 
P_t - L_0 - 10\alpha_1 \log(r_1), & r \leq r_{bp} \\
P_t - L_0 - 10\alpha_1 \log(r_{bp}) - 10\alpha_2 \log(\frac{r}{r_{bp}}), & r > r_{bp} 
\end{cases} (2)
\]

where \( P_t \) is the transmission power, \( L_0 \) is the pathloss in the first meter, \( \alpha_1 \) and \( \alpha_2 \) are the distance power gradients and \( r_{bp} \) is the break point distance. Equations (1) and (2) show that \( P_r(r_1) \) is a Gaussian random variable with mean \( g(r_1) \) and variance \( \sigma \). As the mobile terminal moves away from the AP, it experiences varying data rates at each location due to the random fluctuations in the received signal. Therefore the average spatial PHY throughput \( f(r_1) \) is calculated as

\[
f(r_1) = \sum_i R_i P\{R_i(r_1)\} (3)
\]

---

**TABLE I**

General description of the relationship between receiver sensitivity and throughput

<table>
<thead>
<tr>
<th>Data rate</th>
<th>Receiver Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>( P_1 )</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>decreasing</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( R_M )</td>
<td>( P_M )</td>
</tr>
</tbody>
</table>

**TABLE II**

Manufacturer data sheet parameters for IEEE 802.11ac, two streams using 40MHz bandwidth.

<table>
<thead>
<tr>
<th>( P_t (dBm) )</th>
<th>RSS (dBm)</th>
<th>PHY Data Rate (Mbps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>-62</td>
<td>540</td>
</tr>
<tr>
<td>13</td>
<td>-63</td>
<td>486</td>
</tr>
<tr>
<td>14</td>
<td>-68</td>
<td>405</td>
</tr>
<tr>
<td>16</td>
<td>-70</td>
<td>364.5</td>
</tr>
<tr>
<td>18</td>
<td>-71</td>
<td>324</td>
</tr>
<tr>
<td>19</td>
<td>-77</td>
<td>243</td>
</tr>
<tr>
<td>19</td>
<td>-77</td>
<td>162</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>121.5</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>81</td>
</tr>
<tr>
<td>20</td>
<td>-85</td>
<td>40.5</td>
</tr>
</tbody>
</table>
Fig. 1. General curve shape for PHY through-distance relationship using numerical data from manufacturer datasheet shown in Table II.

where \( P\{R_i(r_1)\} \) is the probability of getting throughput \( R_i \) at distance \( r_1 \) and is given by [11]

\[
P(R_i[r_1]) = \begin{cases} 
1 - \text{erfc}(\frac{r_1 - P_i}{\sqrt{2} \sigma}), & i = 1 \\
\frac{1}{2} \text{erfc}(\frac{r_1 - P_{i-1}}{\sqrt{2} \sigma}) - \frac{1}{2} \text{erfc}(\frac{r_1 - P_i}{\sqrt{2} \sigma}), & i \geq 2 
\end{cases}
\]

Fig. 1 shows that the PHY throughput \( f(r_1) \) shown in (3) is a monotonically decreasing function of distance \( r_1 \).

B. MAC Layer Model

In wireless network, the MAC protocol is responsible for the coordination of multiple terminals over a shared channel. Relays are used to extend the coverage of the access point and to increase the throughput to mobile terminal \( T \). Fig. 2 shows a model of a network with a single relay. Let \( D \) be the distance between the AP and the target terminal \( T \). For a deployed single relay network, the throughput-distance relation between MAC throughput \( S(X) \) for deployment distance \( X \) is shown in Fig. 3. As an example, consider a single relay network shown in Fig. 2, the MAC throughput \( S(D) \) will be bounded by

\[
S(D, 2) \leq \frac{I}{2(\frac{M}{f(r_1)} + \tau_{\text{MAC}})}, \quad \text{where } D = r_1 + r_2
\]

To extend the coverage of the AP to larger areas, additional relays are needed. Each additional relay, will encapsulate the received packet resulting in a larger packet size and a longer slot duration effectively reducing the MAC throughput even further.

C. General Statement of the Problem

Fig. 4 shows the general scenario for a multi-hop network. The first device on the left side is the AP followed by \( N-1 \) relays, \( H_i; i = 1, 2, ..., N-1 \), where \( N \) is the number of hops. The target terminal \( T \) is located at the other end of the network. We assume a packet size of \( M \) bits and a data payload of \( I \) bits resulting in a frame overhead of \( M - I \) bits. We also assume that each relay is visible to its neighboring nodes only. For a

\[
f(r_1) \text{(Mbps)}
\]

\[
r_1 \text{(m)}
\]

where \( f \) is the supported PHY data rate in Mbps. The MAC layer is the delay associated with the processing time of the control frames for channel access and is given by [2]

\[
\tau_{\text{MAC}} = E[X] \cdot E[L] + T_{\text{RTS}} + T_{\text{CTS}} + T_{\text{ACK}}
\]

where \( E[X] \) and \( E[L] \) average number of back-off counts and average length of time for decrease of back off count respectively. Both of these expectations are affected by the window contention size, the probability of successful transmission and the probability of collisions between competing nodes. \( T_{\text{RTS}}, T_{\text{CTS}} \) and \( T_{\text{ACK}} \) are the time durations to transmit the RTS, CTS, and ACK packets for the last successful RTS/CTS handshake. The time durations are defined as the control packet size divided by the transmission rate similar to the relation shown in (6).

From (3), the data rate \( f \) is a function of distance \( r_1 \). For the single relay network shown in Fig. 2, the MAC throughput \( S(D) \) will be bounded by

\[
S(D, 2) \leq \frac{I}{2(\frac{M}{f(r_1)} + \tau_{\text{MAC}})}, \quad \text{where } D = r_1 + r_2
\]
network with $N - 1$ relays using the RTS/CTS protocol, the MAC throughput $S(D, N)$ is bounded by

$$S(D, N) \leq \frac{I}{N\left(\frac{M}{f(r_1)} + \tau_{MAC}\right)}, \quad \text{with } D = \sum_{i=1}^{N} r_i \quad (9)$$

where $D$ is the distance from the AP to target terminal $T$. Equation (9) indicates that the MAC throughput at distance $D$ is a function of the PHY throughput of the first link $f(r_1)$ and the number of hops $N$.

### III. PERFORMANCE ANALYSIS

#### A. Intermediate Distance Between Relays and Access Point

We start the analysis by determining the intermediate physical separation distances between the AP and relays to maximize the MAC throughput for a target terminal located at a distance $D$ from the AP.

**Theorem 1**: Given a distance $D$ in a wireless data network with $N - 1$ relays, the optimal intermediate distance between the relays and the AP is

$$r_1 = r_k = \text{ for } 2 \leq k \leq N \quad (10)$$

**Proof**: From (3) and (4), the PHY throughput $f(r_1)$ is a monotonically decreasing function of distance $r_1$. Mathematically this is represented as $f'(r_1) \leq 0$. In order to carry the traffic through a network with $k - 1$ relays, all relay nodes must at least carry the PHY throughput of the AP. That is $f(r_k) \leq f(r_1); 2 \leq k \leq N$.

Since $f(r_1)$ is a monotonically decreasing function, this implies that $r_k \leq r_1$. Therefore, the distance $D$ separating the AP and $T$ is bounded by $D \leq kr_1$. From $f'(r) \leq 0$, we require the minimum value of $r$ that maximizes the PHY throughput. From the distance bound we have $r_1 \geq \frac{D}{K}$ with a minimum value of $\frac{D}{k}$. This implies that all the intermediate $r_k$ distances are equal, resulting in (10).

For a network with $N - 1$ relays, from (10) we have $r_1 = \frac{D}{N}$. Then the upper bounds for $S(D)$ can be represented as a function of distance $D$ and the number of hops $N$ as

$$S(D, N) \leq \frac{I}{N\left(\frac{M}{f(r_1)} + \tau_{MAC}\right)} \quad (11)$$

For a general monotonically decreasing function, this provides the maximal bounds on the MAC throughput as a function of target distance $D$ and the number of hops $N$. Fig. 5 shows the optimal MAC throughput bound in (11) for different number of relays. In all cases of Fig. 5, the AP and relays are equidistant from each other.

#### B. Optimal MAC Throughput Bounds and Minimum Number of Relays

Given the MAC throughput bound in (11), we determine the maximum achievable throughput at a distance $D$ and the required number of relays to reach that throughput.

**Theorem 2**: In relay network for any given distance $D$, there is an optimal number of relays $N - 1$ that can attain the optimal MAC throughput $S(D)$ given by

$$S_{\text{max}}(D) = \frac{I}{Z_{N-1} \leq D \leq Z_N} \quad \text{where } Z_{N-1} \leq D \leq Z_N \quad (12)$$

and $Z_N = D|_{S(D,N-1)=S(D,N)}$ for $N = 1, 2, 3, ..., k$.

**Proof**: The Theorem can be proven by induction. We present a base case and an inductive step to prove it. From (8), the MAC throughput for the direct connection is $S(D, 1)$ and the MAC throughput for a single relay network is $S_2(D)$. They can be represented as

$$S(D, 1) = \frac{I}{\tau_{PHY} + \tau_{MAC}}$$

$$S(D, 2) = \frac{I}{2\left(\tau_{PHY} + \tau_{MAC}\right)} \quad (13)$$

The PHY throughput function $f$ is a monotonically decreasing function. From Fig. 5, $S(D, 1)$ decreases faster than $S(D, 2)$. However $S(D, 1)$ has a larger initial values than $S(D, 2)$. Therefore $S(D, 1) \cap S(D, 2)$ at distance $D = Z_1$. For $D \leq Z_1$ The MAC throughput for the direct connection to the AP $S(D, 1)$ is greater than the MAC throughput for the single relay $S(D, 2)$. Therefore we do not need any relays and the upper bound is $S(D, 1)$. For $D > Z_1$ we need a single relay and the upper bound is $S(D, 2)$ then the optimal performance bounds are

$$S_{\text{max}}(D) = \begin{cases} S(D, 1) = \frac{I}{\tau_{PHY} + \tau_{MAC}} & \text{if } D \leq Z_1 \\ S(D, 2) = \frac{I}{2\left(\tau_{PHY} + \tau_{MAC}\right)} & \text{if } D > Z_1 \end{cases}$$

$$S_{\text{max}}(D) = \begin{cases} S(D, 1) = \frac{I}{\tau_{PHY} + \tau_{MAC}} & \text{if } D \leq Z_1 \\ S(D, 2) = \frac{I}{2\left(\tau_{PHY} + \tau_{MAC}\right)} & \text{if } D > Z_1 \end{cases} \quad (14)$$
Consider the case for \( k - 1 \) relay and \( k \) relay networks, from (8) we have

\[
\begin{align*}
S(D, k) &= \frac{I}{k\left(\frac{I}{j^2} + \tau_{\text{MAC}}\right)} \\
S(D, k + 1) &= \frac{I}{(k+1)\left(\frac{I}{j^2} + \tau_{\text{MAC}}\right)} \quad (15)
\end{align*}
\]

\( S(D, k) \) has a larger initial values and has a faster rate of decay than \( S(D, k + 1) \). Therefore \( S(D, k) \cap S(D, k + 1) \) at \( D = Z_k \). For \( D \leq Z_k \) The MAC throughput with \( k - 1 \) relays is greater than the MAC throughput for a network with a \( k \) relays. Therefore we do not need any relays and the upper bound is \( S_k(D) \). For \( D > Z_k \) we need \( k \) relays and the upper bound is \( S_{k+1}(D) \) and the optimal performance bounds are

\[
S_{\text{max}}(D) = \begin{cases} 
S(D, k) = \frac{I}{k\left(\frac{I}{j^2} + \tau_{\text{MAC}}\right)} & D \leq Z_k \\
S(D, k + 1) = \frac{I}{(k+1)\left(\frac{I}{j^2} + \tau_{\text{MAC}}\right)} & D > Z_k
\end{cases} \]

(16)

By induction, we can conclude that for \( k \geq 1 \)

\[
S_{\text{max}}(D) = \frac{I}{k\left(\frac{M}{j^2} + \tau_{\text{MAC}}\right)} \quad \text{with} \quad Z_k - 1 \leq D \leq Z_k \]

(17)

where \( Z_k = D|_{S(D, k) = S(D, k+1)} \).

In summary, given \( N - 1 \) relays, the optimal relay deployment to maximize the throughput for a target receiver located \( D \) away from the AP is \( r_k = D/N; \ 1 \leq k \leq N - 1 \). To determine the number of relays for optimal deployment for a given \( D \), we use the distance bound in (12).

**C. Application to 802.11ac**

In this subsection, we apply the results of our analysis to the IEEE 802.11ac standard. We consider two deployment scenarios. In both scenarios we assume a bandwidth of 40MHz and two spatial streams in our analysis. The first is to
maximize the deployment spatial throughput up to a desired coverage distance. The second is to optimize the deployment to deliver the maximum achievable throughput for a target located at a specific distance $D$. We assume the data payload $I = 8184$ bits and a frame header overhead of $H = 400$ bits. The channel access overhead from the MAC layer processing $\tau_{MAC} = 109\mu$s based on the calculation provided in [2].

Let us assume the target is located at a distance $D = 180m$. Given the manufacturer data sheet in Table II and the pathloss model in (3) and (4) we obtain the PHY throughput distance relationship shown in Fig. 1. We consider the first scenario to maximize the spatial throughput up to $D$. The addition of relays will increase the packet size from the encapsulation process. The increase of the packet size will increase the slot duration and will reduce the MAC throughput. Therefore to maximize the spatial throughput we must use the minimum number of relays. The relay selection criterion is based on the performance bounds shown in Fig. 5. The bound in Fig 5 is a function of the number of relays and the distance $D$. For each $S(D, N)$, After certain distance, the available data rate will be fully occupied by the header and therefore no data can be transmitted. Therefore we need to add a relay to increase the coverage beyond that threshold distance. One approach to selecting the threshold distance is to consider minimum allowable data rate for the target terminal. We assume the minimum allowable throughput at distance $D$ to be $5Mbps$. From Fig. 5, the curve that satisfies this minimum criteria is the single relay $(N = 2)$ curve. The optimal deployment is a direct result of (10) where the relay is placed at a midpoint distance of $90m$ from the AP. Fig. 6 shows the optimal deployment MAC throughput-distance relation for this single relay network case.

We now consider the second scenario that maximizes the MAC throughput at $D = 180m$. From Fig. 5, we see that the maximum throughput at a distance of $180m$ is $14.18Mbps$ using 2 relays $(N = 3)$. Using (7), The relays are placed $60m$ apart. The 3 relays deployment is shown in Fig. 6. The results in Fig. 6 reveal that the 3 relay network delivers higher throughput than the single relay network at distances greater than $150m$. This holds for the desired distance at $180m$.

IV. CONCLUSION

In this paper, we presented a novel approach to obtain the optimal performance bounds for a multi-hop multi-rate wireless data network. First, we determined the optimal relay placements for a target terminal located at a distance $D$ away from the access point. Second, for a general analytical PHY layer throughput model, we determined the maximum achievable MAC throughput as a function of the number of relays for a target located at distance $D$. In the analysis, we considered the IEEE 802.11 RTS/CTS MAC protocol. Finally, we determined the required number of relay nodes to maximize the MAC throughput for the target receiver. To demonstrate the analysis, we considered two deployment scenarios. In the first scenario we maximize the MAC throughput for a desired coverage distance. In the second scenario we maximize the MAC throughput for a specific target distance.

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